Divide-and-conquer Algorithms for Counting Paths using Zero-suppressed Binary Decision Diagrams

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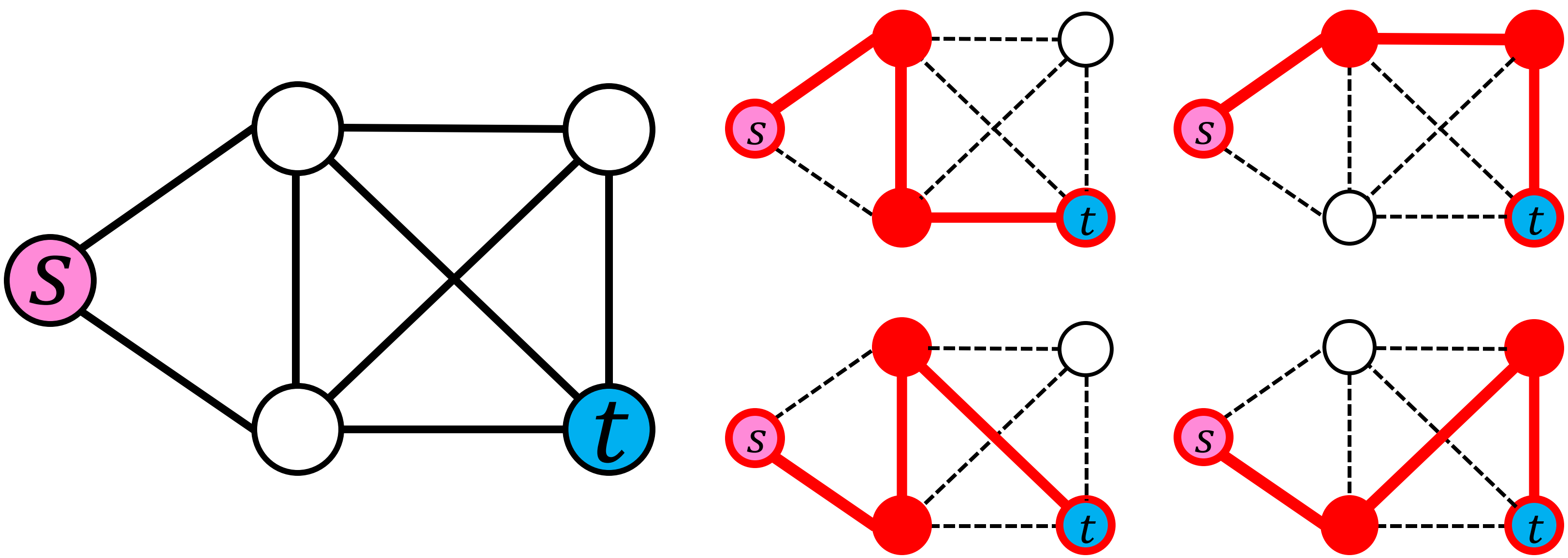
August 2, 2024

WAAC2024

Path-counting problem

Input: A graph $G(V, E)$, two terminals $(s, t) \in V$, and a nonnegative integer ℓ

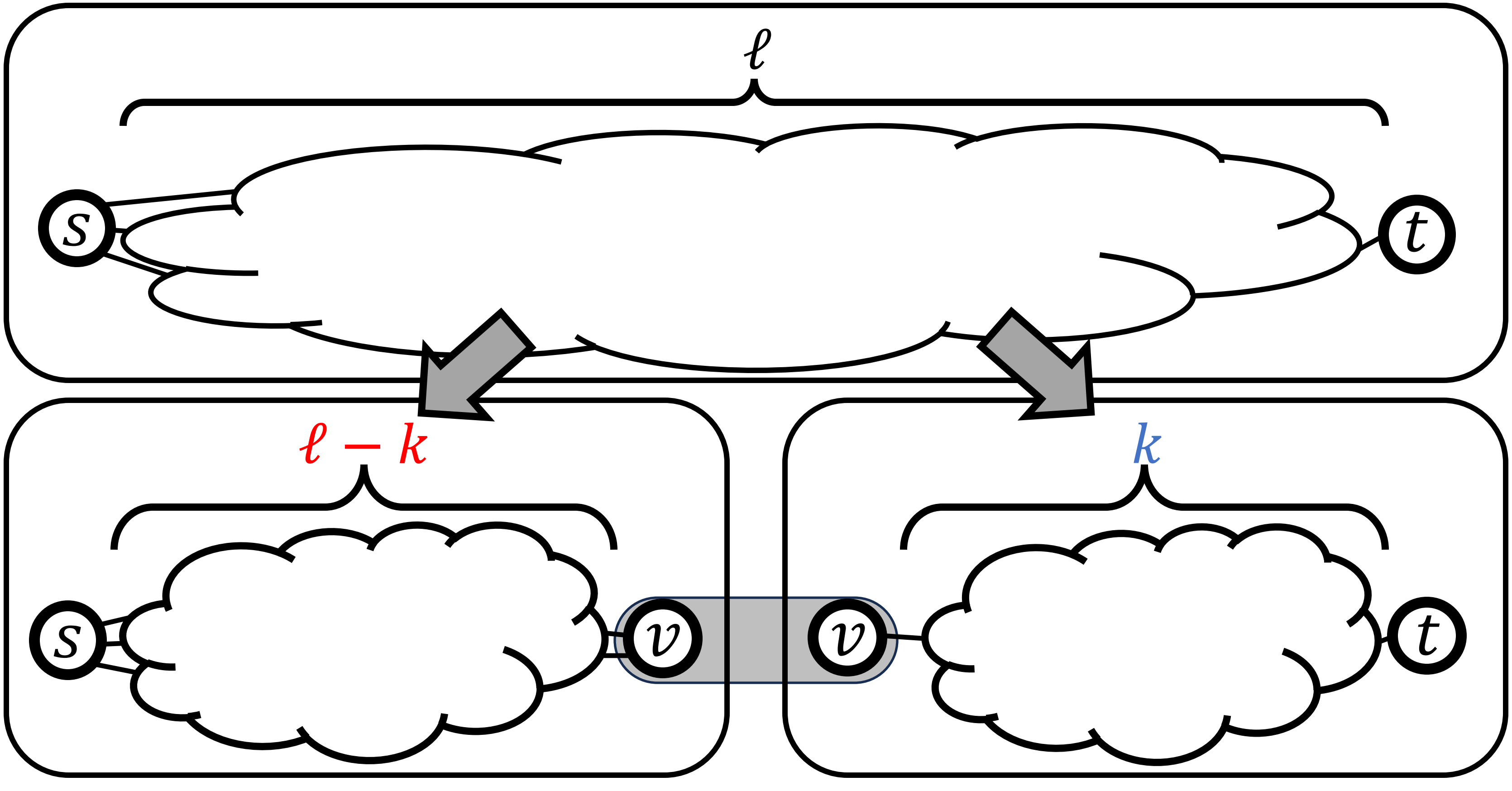
Question: How many s-t paths of length ℓ exists in G ?



Lists of s-t paths of length 3

Overview of our approach

We divide the problem into sub-problems



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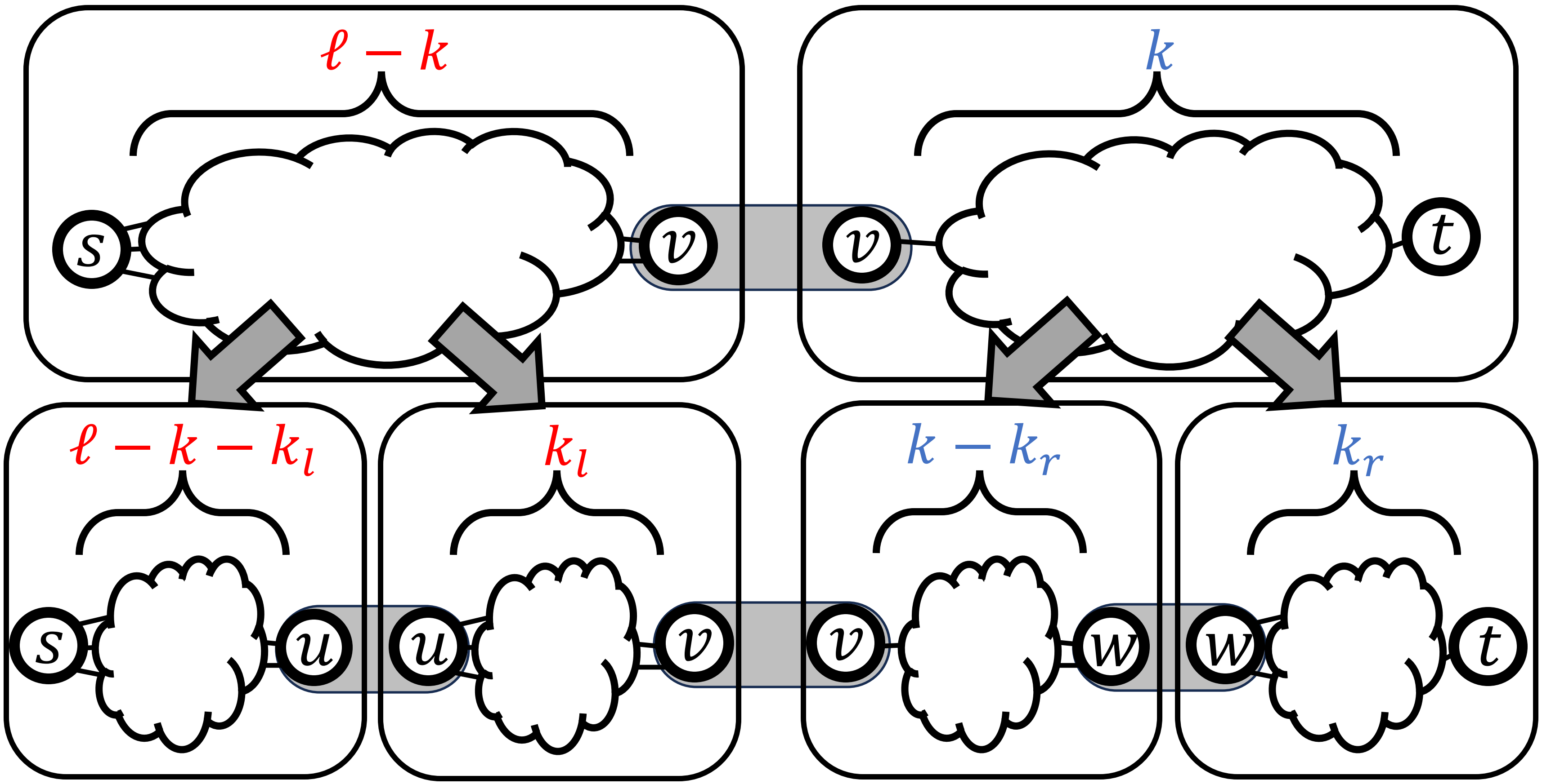

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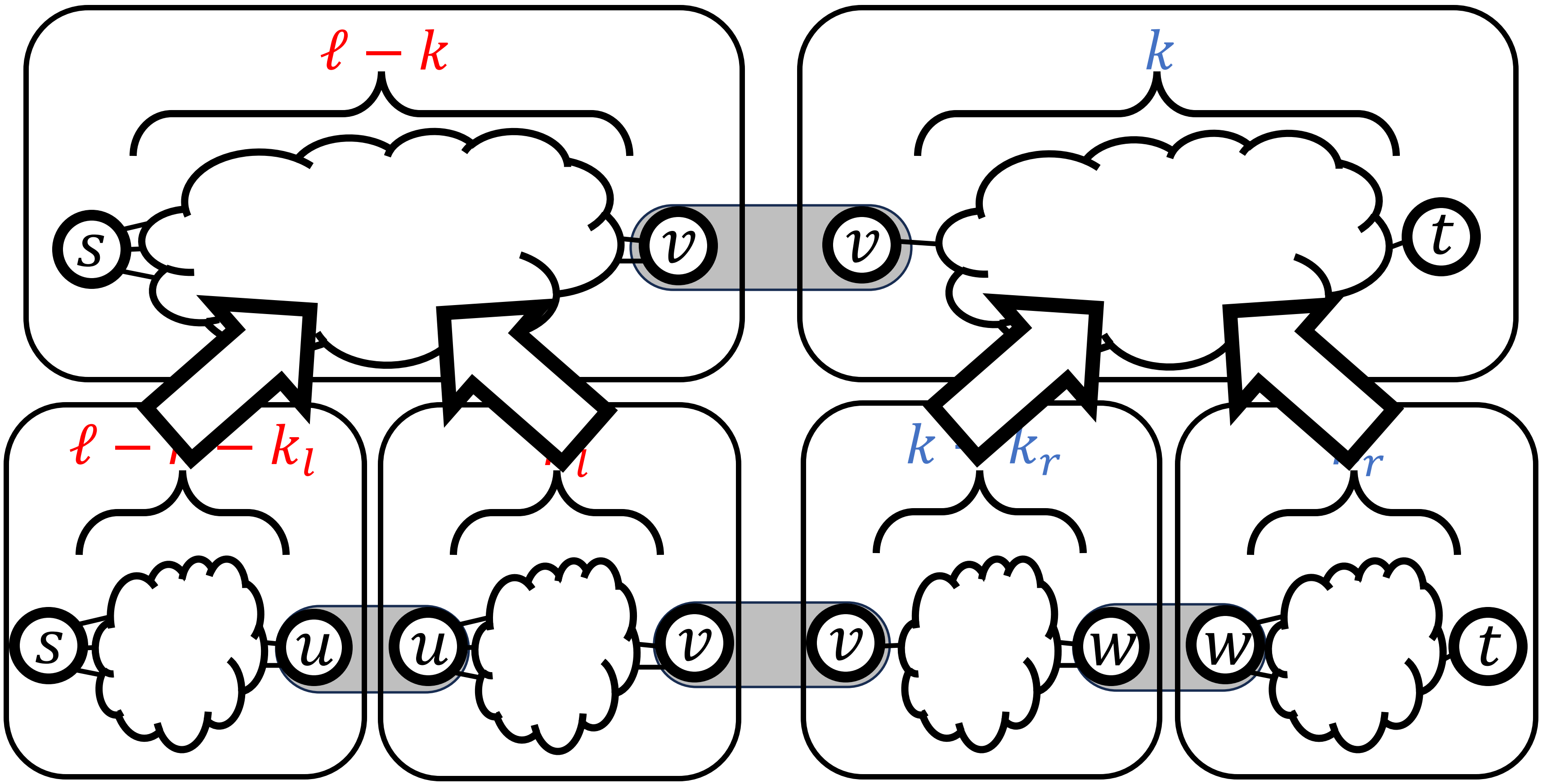
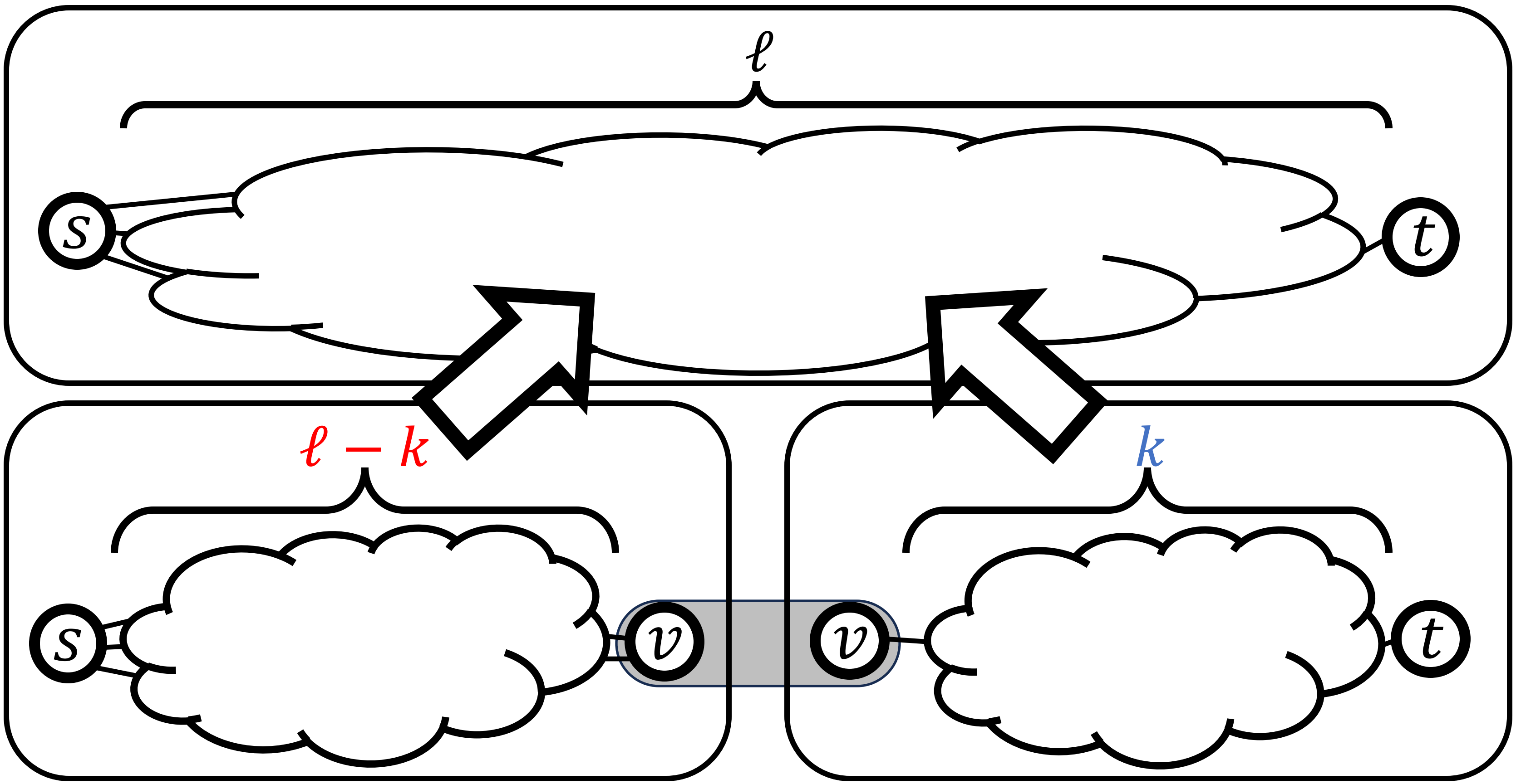


Multiple the number of paths

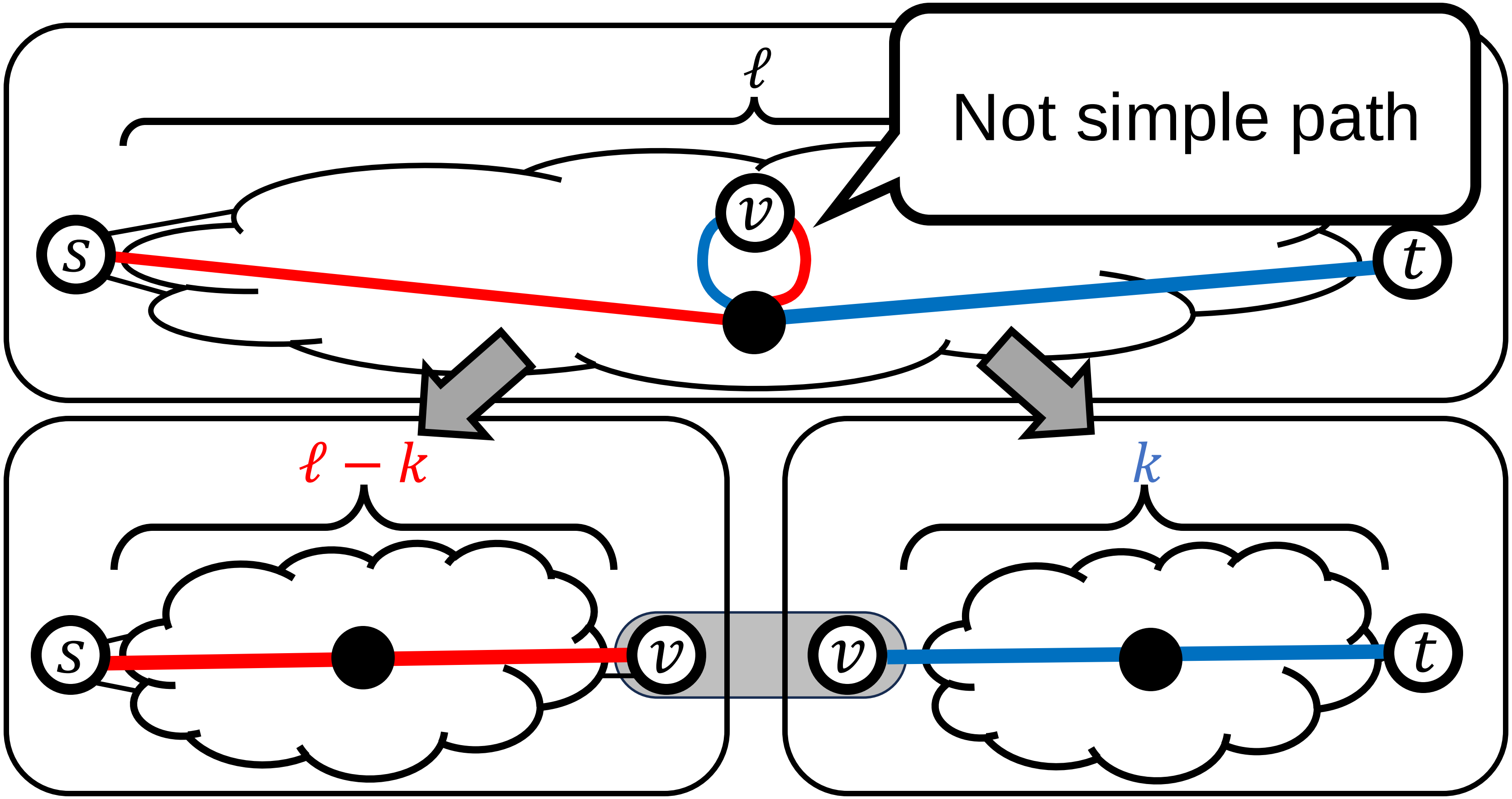
Overview of our approach

We divide the problem into sub-problems



Multiple the number of paths

Overview of our approach



Do not connect overlapping paths
→ **Multiplication is imperfect** for connecting paths

Our Results

✓There are performance differences due to the division length

i. Half: $\left(\left\lceil\frac{\ell}{2}\right\rceil, \left\lfloor\frac{\ell}{2}\right\rfloor\right)$

ii. Edge by edge: $(\ell - 1,1)$

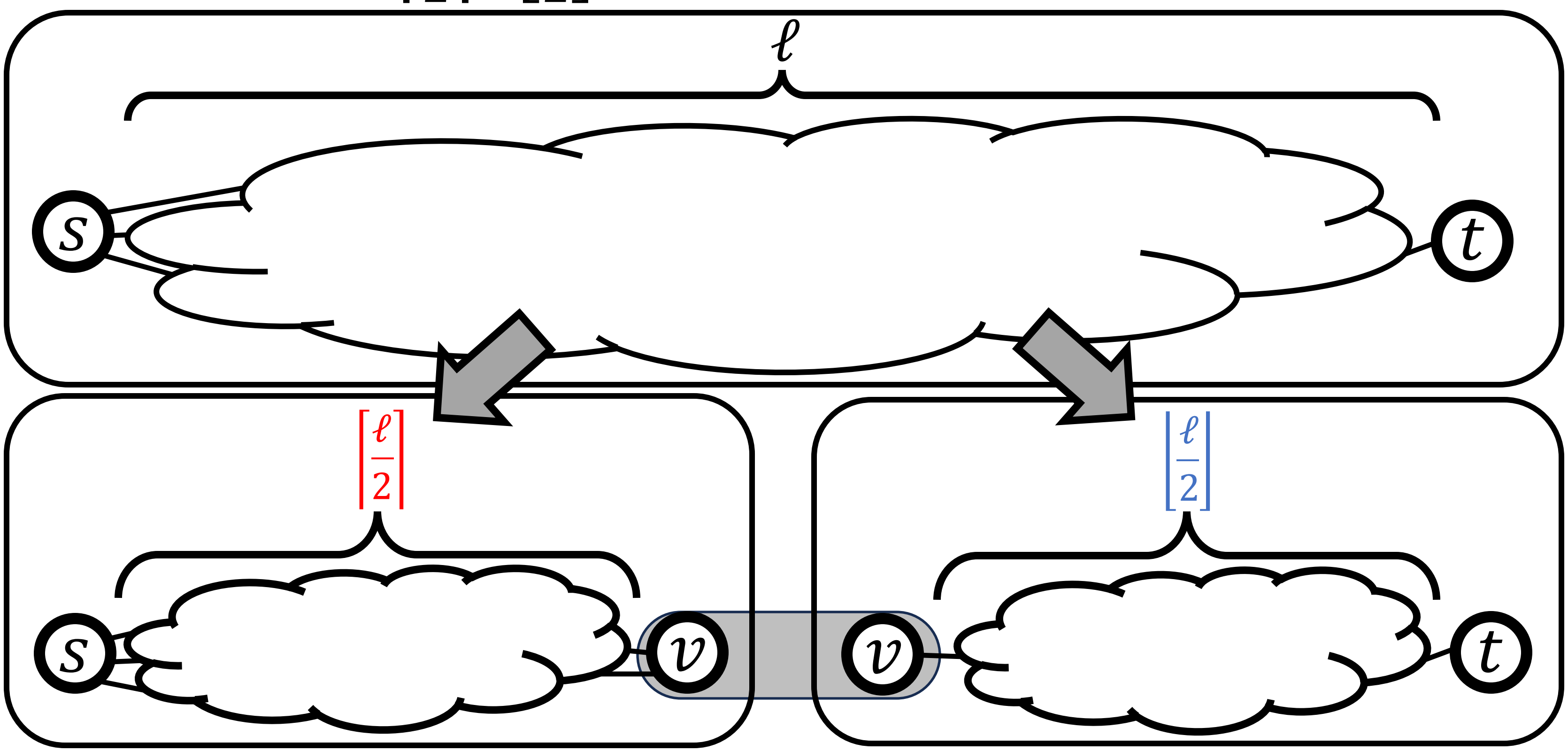
iii. Hybrid of i and ii

✓Implement **ZDD-based** divide-and-conquer algorithm for the problem.

Our Results

✓There are performance differences due to the division length

i. Half: $(\lceil \frac{\ell}{2} \rceil, \lfloor \frac{\ell}{2} \rfloor)$



Our Results

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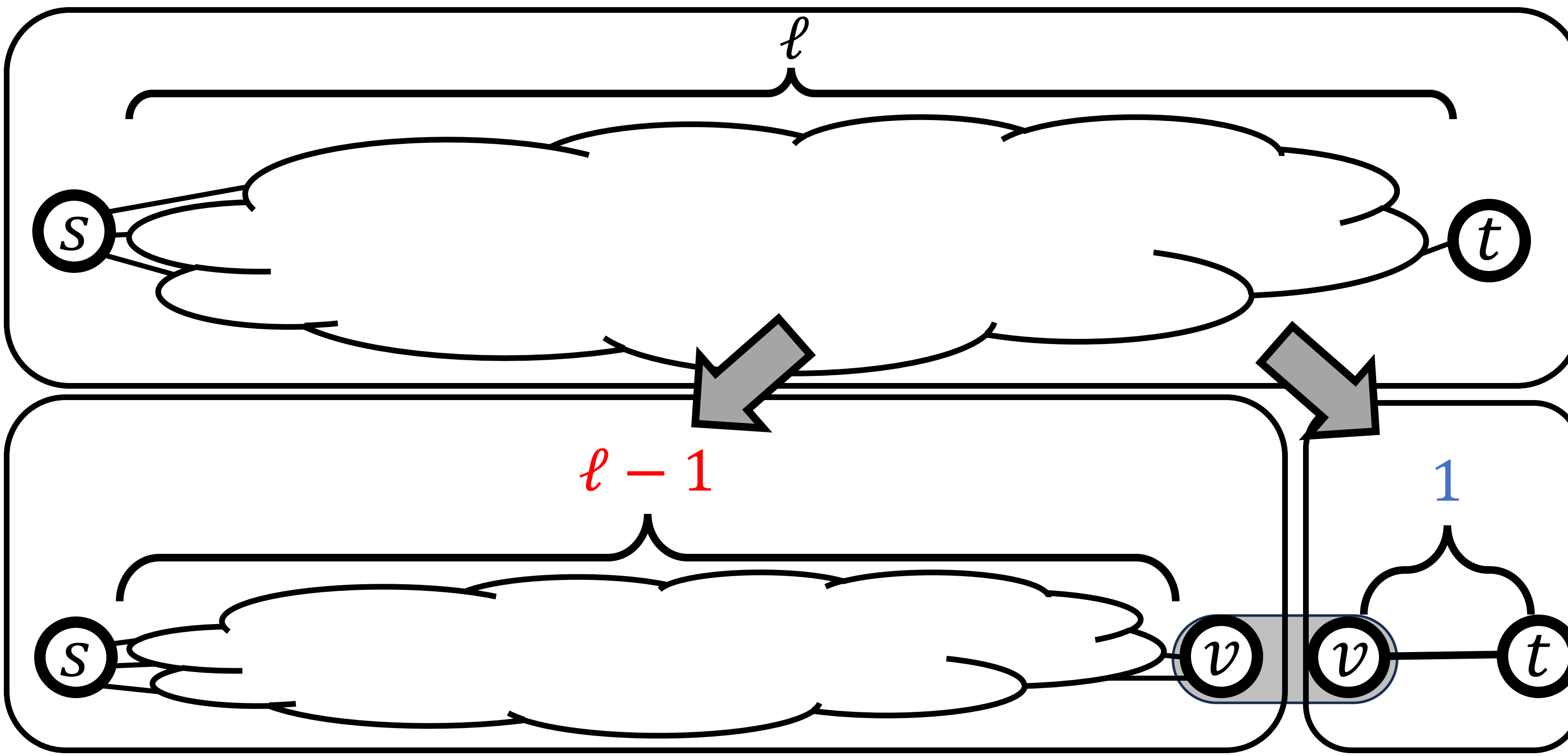
ii. Edge by edge: $(\ell - 1, 1)$



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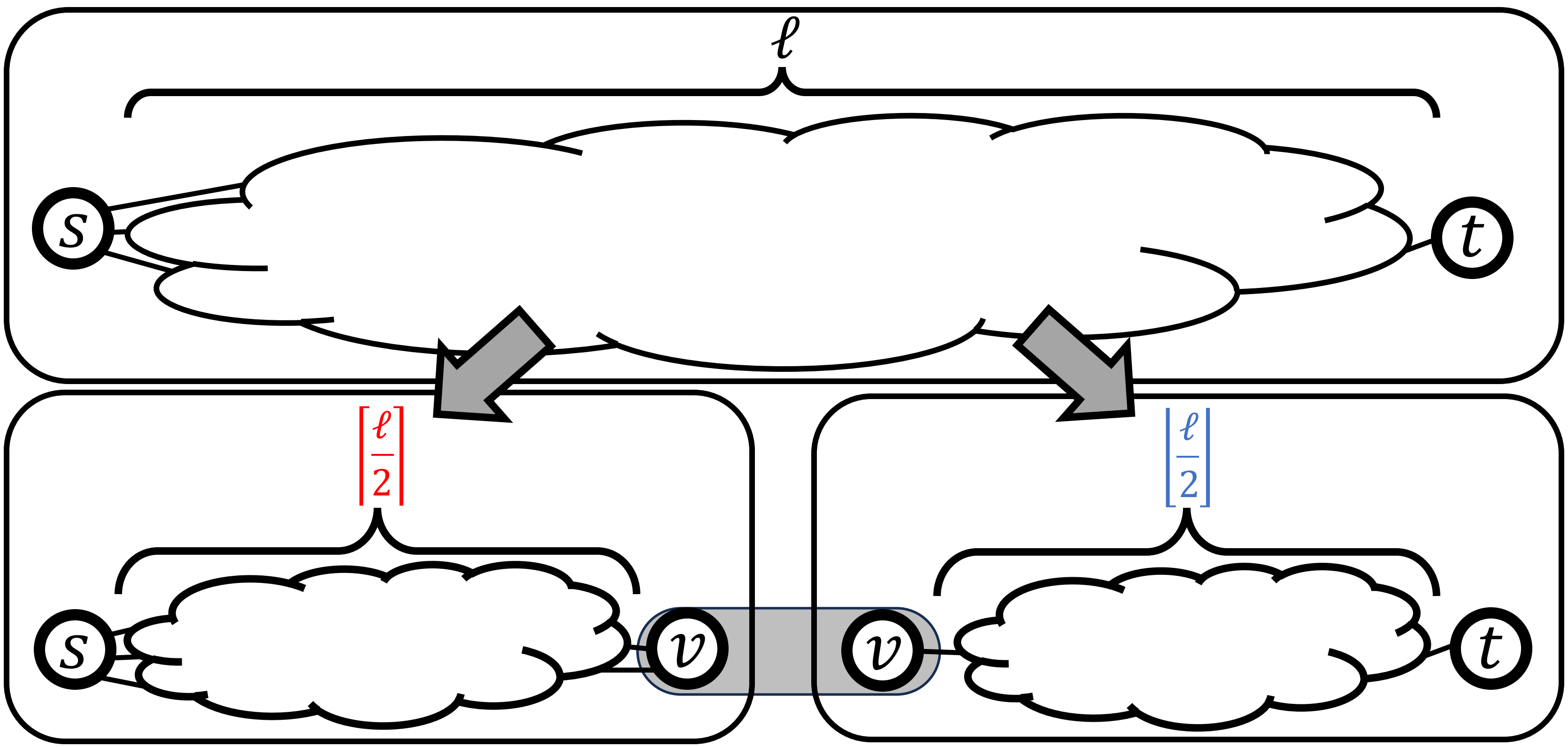

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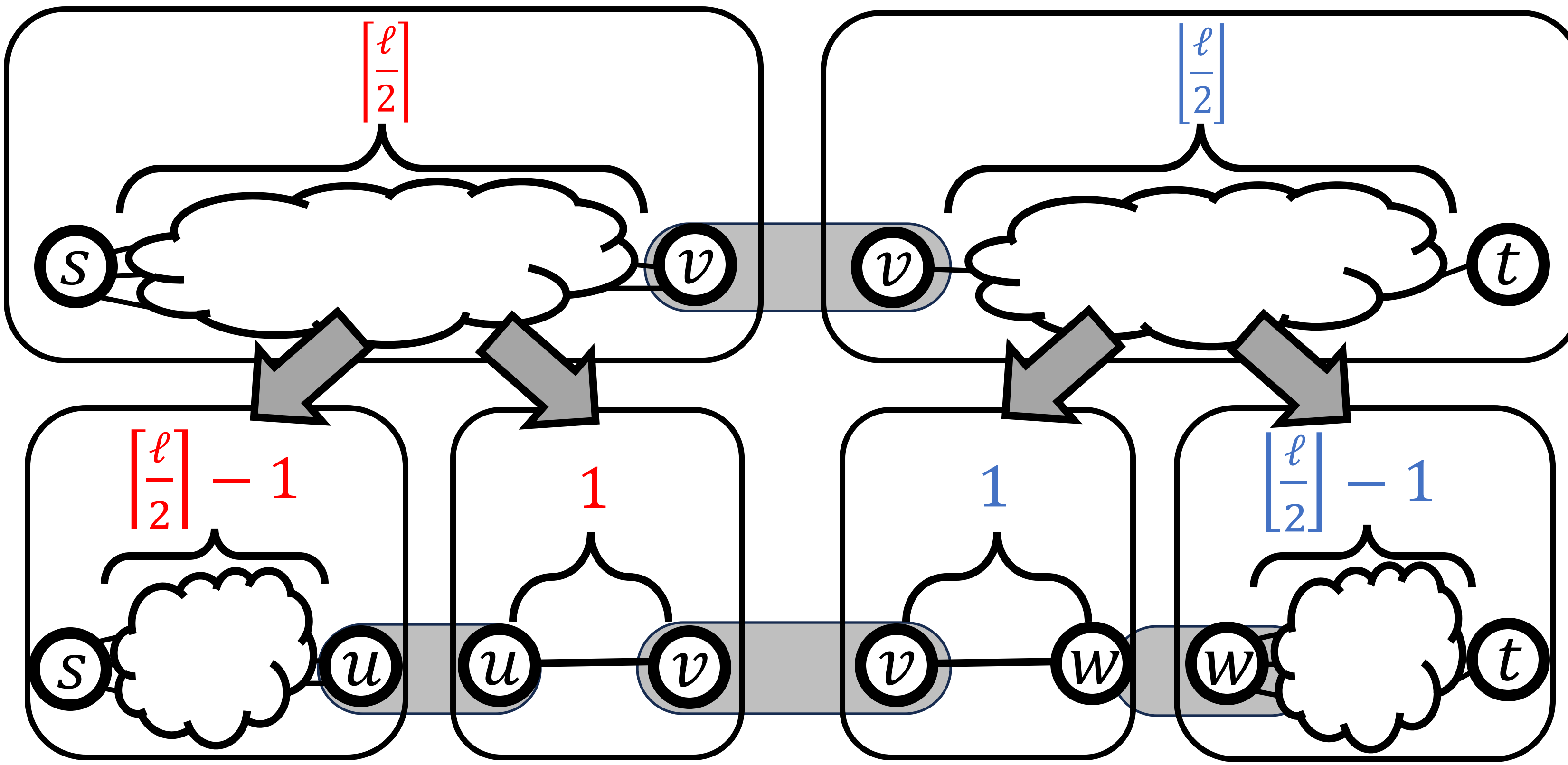


Our Results

✓There are performance differences due to the division length

i. Half: $\left(\left\lceil\frac{\ell}{2}\right\rceil, \left\lfloor\frac{\ell}{2}\right\rfloor\right)$
ii. Edge by edge: $(\ell - 1, 1)$
iii. Hybrid of ① and ②

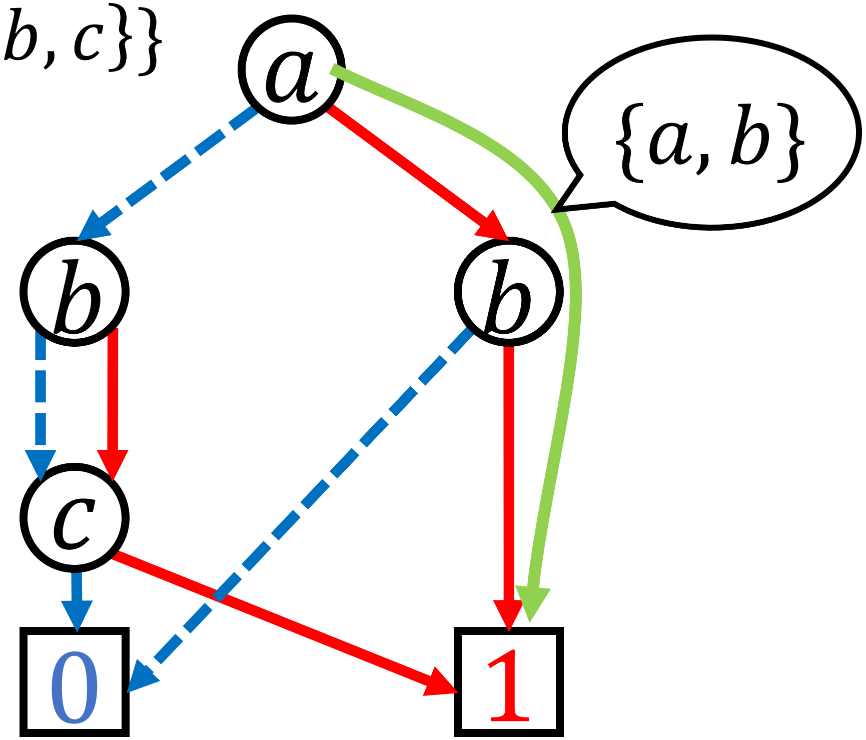
✓Implement **ZDD-based** divide-and-conquer algorithm for the problem.

ZDDs

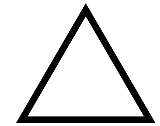
Select an element
Do not select an element

ZDDs : Data structures representing families of sets compactly as directed graphs

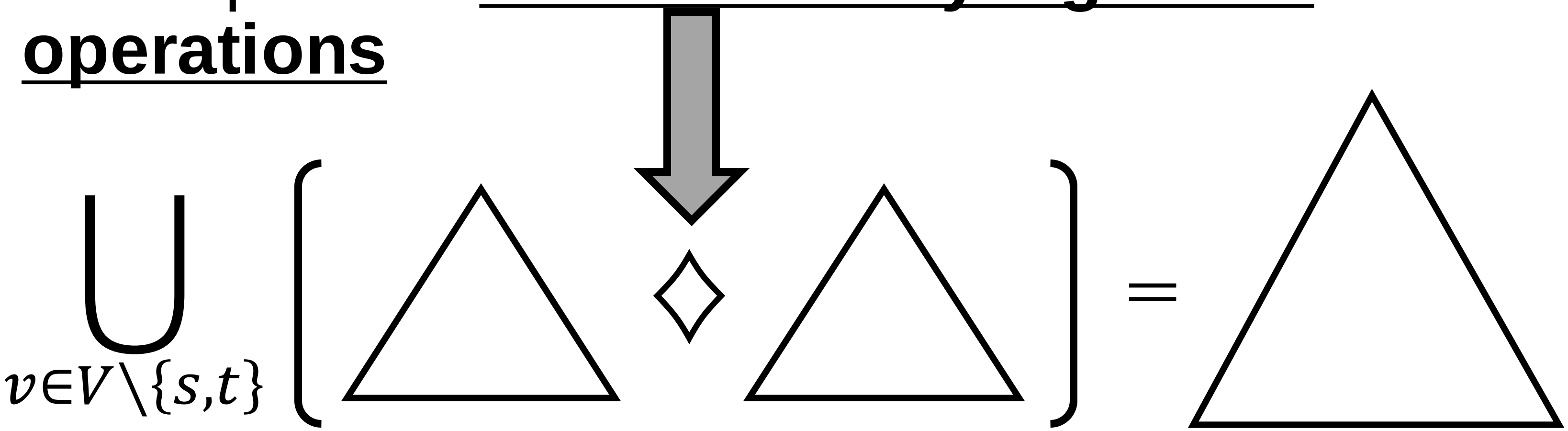
[Example] Following ZDD representing
$S = \{\{c\}, \{a, b\}, \{b, c\}\}$



ZDDs



ZDDs provide **efficient family algebraic operations**



Not suitable for connecting **only non-overlapping paths**

→ **We propose a new operation "disjoint join"**

Computational experiments

Input: 100 instances (provided by ICGCA2023)

Timeout: 600 seconds / instance

Environment

CPU：Intel Xeon CPU E5-2643 v4
(3.40 GHz, 24 cores)

OS：CentOS 7.9 **Memory**：512GB

Library Our algorithms: SAPPOROBDD
Mate-frontier method: TdZdd

Running time

- : timeout (600s)

	Running time[s]			
No.	Half	Edge	Hybrid	Mate-Frontier
022	0.69	36.28	0.49	-
025	-	-	-	212.25
061	1.67	50.84	1.38	-
073	2.53	145.74	1.50	-
085	-	119.57	-	-
089	171.23	-	137.13	-

There are performance differences due to the divide length

Running time

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No.	**Half**	**Edge**	**Hybrid**	**Mate-Frontier**
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Hybrid calculates the fastest on half of instances.

Running time

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	Running time[s]			
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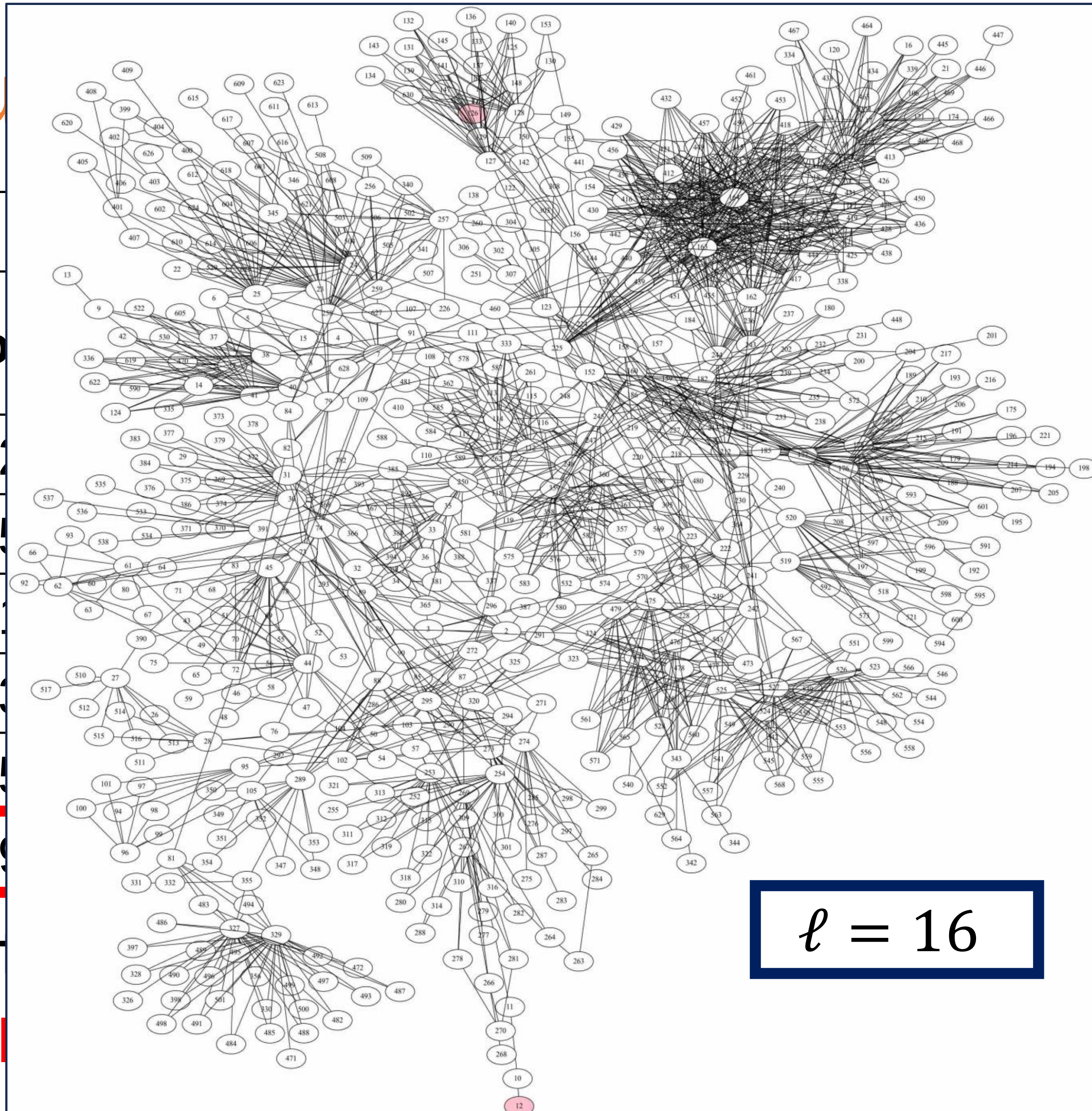
Half and **Hybrid** are solvable for

higher $|E|/|V|$ ratio with **clique-like** structures

Ru

out (600s)



No	er
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Running time

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Edge is solvable for

lower $|E|/|V|$ ratio, which are characterized as **sparse** graphs.

R (600s)

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02
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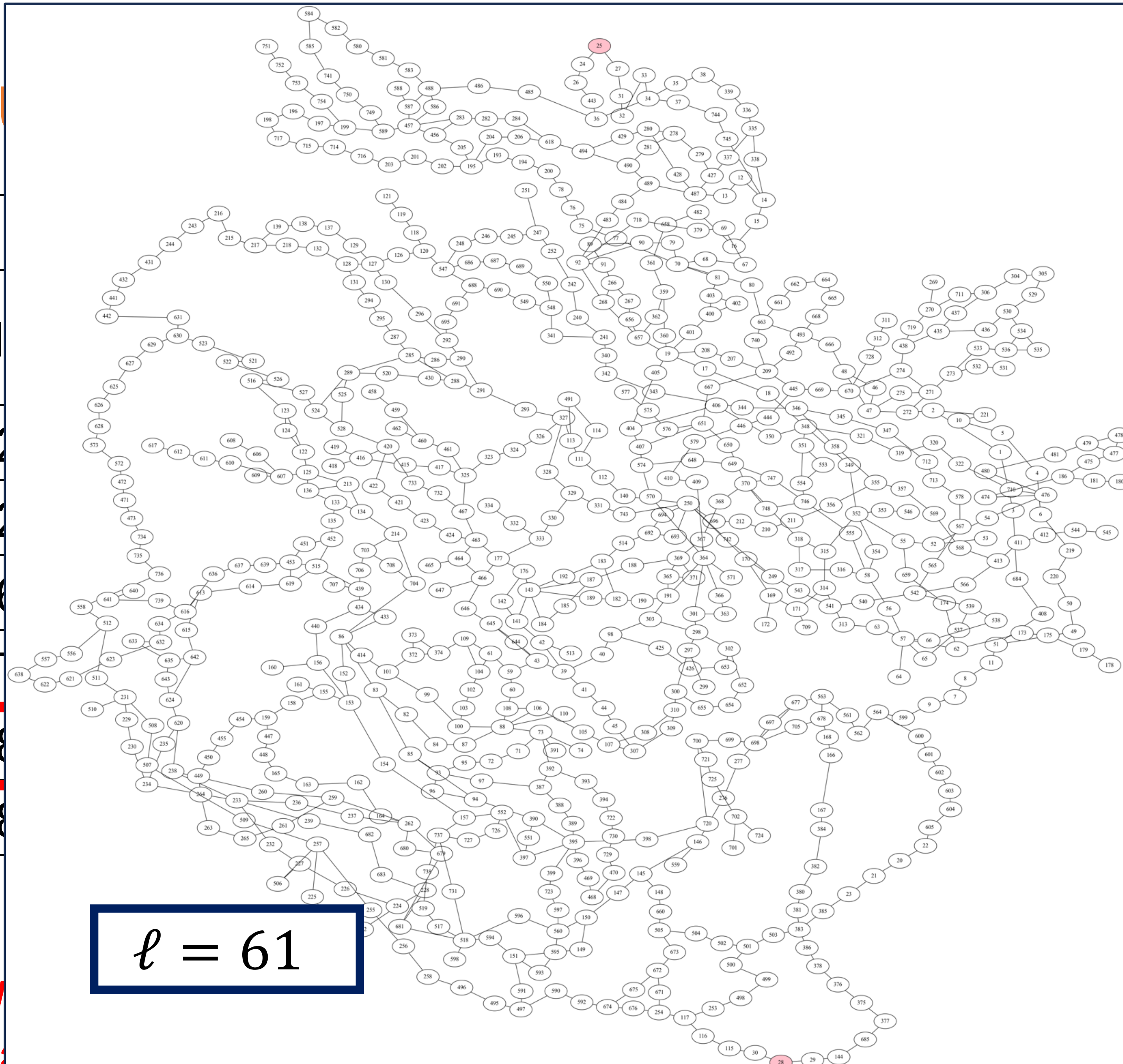

Ed

low as

sp graphs

Running time

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It is hard to calculate **long path** in **densely** graph

R… (600s)

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02
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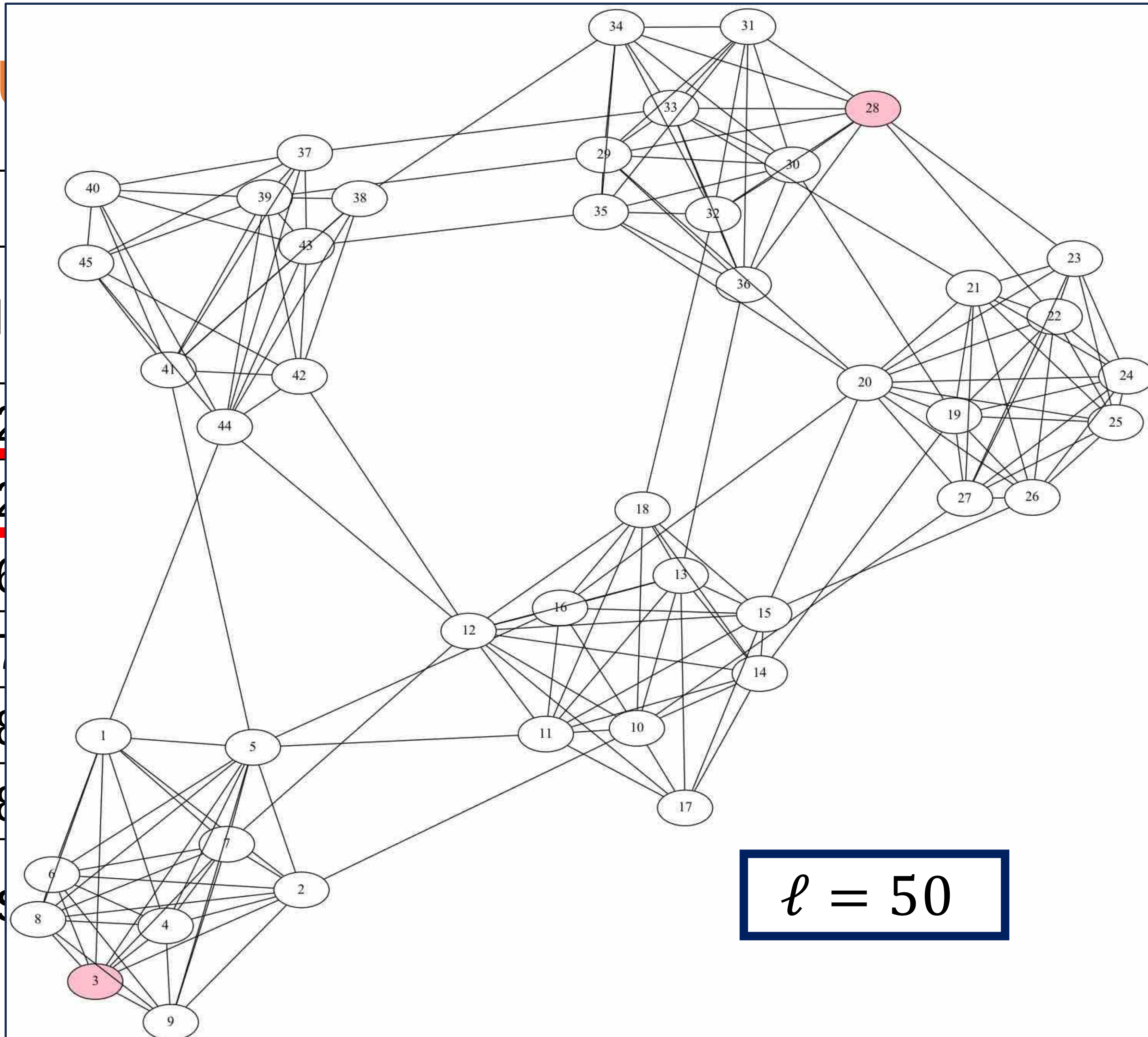

It is … ph

Conclusion

✓Implement **ZDD-based** divide-and-conquer algorithms for **path-counting problem**

✓Observe the types of graph structures corresponding to division length

Future work:

More experiment with various characteristics (pathlength, pathwidth, maximum clique size, etc.)